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CP asymmetries in decays of the $D^0 - \bar{D}^0$ system revisited

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Abstract. CP violation in neutral D meson decays to the CP eigenstates and non-CP eigenstates is studied systematically within the framework of the Cabibbo-Kobayashi-Maskawa model. The nonleptonic two-body decay processes and the decay processes with semileptonic tagging are discussed in detail and the upper bounds of the direct and indirect CP violation in these decay processes are obtained. A method to measure the mixing parameter $\tilde{\epsilon}$ and to separate the direct and indirect CP violation in the decay processes with semileptonic tagging are also discussed.

1 Introduction

The charm physics have been studied extensively since the discovery of charmed particles in 1976 [1–3]. Up to now, although the precision of the measurements needs to be improved, we have a large number of experimental data on charm decays especially in the neutral D meson sector [4]. The rich variety of available charm decay modes offers a possibility to sudy decay mechanism of the charm hadrons and to test the different theoretical methods.

The $D^0 - \bar{D}^0$ mixing is closely connected to CP violation in neutral D meson decays. The study of the $D^0 - \bar{D}^0$ mixing is not only complementary to our CP violation knowledge on the $K^0 - \bar{K}^0$ systems, but also important for testing the standard model and probing new physics beyond the standard model [5]. The mixing rate is defined as

$$r_D = \frac{Number\ of\ D^0\ decaying\ as\ \bar{D}^0}{Number\ of\ D^0\ decaying\ as\ D^0}$$

In the standard model r_D is expected to be very small. The short distance contribution to $D^0 - \bar{D}^0$ mixing is via box diagrams and is expected to be negligiblly small because the GIM cancellation in the box diagram is almost perfect. The theoretical estimate for the long distance contribution is controversial for different methods. The early estimate gives $r_D \sim 10^{-4}$ [6], and $\sim 10^{-6}$ [7]. Later on, Georgi et al. [8] analyze it using Heavy Quark Effective Field Theory (HQEFT) and claim that the long distance contributions may be considerably smaller than the estimates in [6,7]. A recent estimate shows that the short and long distance contributions are in the same order of magnitude and $r_D \sim 10^{-10} \sim 10^{-9}$ [9]. In experiment, observation of $r_D > 10^{-4}$ will imply the existance of new physics. The present upper bounds for $D^0 - \bar{D}^0$ mixing are $r_D < 4.7\times10^{-3}$ (FNAL E791) and $r_D < 3.7\times10^{-3}$(FNAL E691) [10].

Since the long distance contribution and the effect of final-state interactions can not be calculated reliablly at present, the theoretical calculation for $D^0 - \bar{D}^0$ mixing and the CP violation in the charm system is not satisfactory. So systematic study of the phenomenology $D^0 - \bar{D}^0$ mixing and CP violation in charm system becomes necessary and important for both further theoretical study and experimental efforts at the high-luminosity fixed target facilities, the forthcoming B-meson factories and the proposed τ-charm factories.

The phenomenology of $D^0 - \bar{D}^0$ mixing and CP violation in neutral D meson decays was first discussed by Pais and Treiman, Bigi and Sanda [11] and have been further studied afterwards [12]. But there are still many uncertainties. Further studies are definitely needed. In this paper, we shall refine the phenomenology of $D^0 - \bar{D}^0$ mixing and CP violation in neutral D meson decays. In our method there are several features which are different from the early discussions

(a) Our caculation contains the effects of CP violation caused by mixing parameter $\tilde{\epsilon} = |p|^2 - |q|^2$. Sometimes, the CP violation term proportional to $\tilde{\epsilon}$ is also important. Especially for the decay processes of the coherent $(D^0\bar{D}^0)$ pair using semileptonic tagging, we have the CP asymmetry $\mathcal{A}_{CP}^{(-)}(f, \ell^+X^-) \approx 2\tilde{\epsilon}$. In some early discussions in the literature the overlooked processes might be the most promising ones to observe CP violation in neutral D meson decays if $|\tilde{\epsilon}| \sim 10^{-3}$.

(b) We analyse in detail the indirect CP violating asymmetry arising from the interplay between mixing and decay for the $D^0 - \bar{D}^0$ system decaying into CP eigenstates. It turns out that this indirect CP asymmetry is $-xsin\phi_f + y\tilde{\epsilon}cos\phi_f$. In the early discussions, the term $y\tilde{\epsilon}cos\phi_f$ is often neglected (since taking approximation $|p|^2 = |q|^2$), but for the term $-xsin\phi_f$, the magnitude of $sin\phi_f$ is uncertain (sometimes taking $sin\phi_f = 1$ is not correct). So we can

not always omit the term $y\tilde{\epsilon}cos\phi_f$.
(c) Since the difficulty both in theoretical calculation and experimental measurement, the strong phase-shift $\delta_f = \delta_1 - \delta_2$ (where the decay amplitude is written as $A(f) = G_1 T_1 e^{i\delta_1} + G_2 T_2 e^{i\delta_2}$)is unknown at present [13]. We use extremum method to estimate the upper bounds of direct and indirect CP violation. These upper bounds of CP asymmetries are more reliable and thus useful for designing experiments.

The outline of this paper is as follows. The generic time-integrated CP asymmetry formulas for the decay of the neutral $P^0 - \bar{P}^0$ meson system are represented in Sect. 2. Using these CP asymmetry formulas given in Sect. 2, we discuss systematically the $D^0 - \bar{D}^0$ mixing and CP violation in neutral D meson decays into CP eigenstates and non-CP eigenstates in Sect. 3. Our calculated results will be discussed in Sect. 4.

2 Generic time-integrated CP asymmetry formulas

In this section, we will give the generic time-integrated CP asymmetry formulas for the decay of the neutral $P^0 - \bar{P}^0$ meson system. These phenomenological CP asymmetry formulas are model independent and analytically accurate since there are no special assumption in our derivation. So it is universally applicable for the decay of the different neutral $P^0 - \bar{P}^0$ meson system. These formulas are firstly discussed by Pais and Treiman; Bigi and Sanda [11], then developed by many authors [14]. All of the formulas are essentially the same, but different people emphasize different aspects. In this paper, we use our expressions for convenience.

2.1 CP asymmetry for the decay of the neutral meson system

Take the phase convention as $CP|P^0>=|\bar{P}^0>$ and assume CPT invariance, then the mass eigenstates of P^0 and $\bar{P}^0$ mesons can be written as

$$\begin{aligned} |P_L> &= p|P^0> + q|\bar{P}^0> \\ |P_H> &= p|P^0> - q|\bar{P}^0> \end{aligned} \tag{2.1}$$

with the eigenvalues $\lambda_{L,H} = m_{L,H} - \frac{i}{2}\gamma_{L,H}$, where the subscript indicates light or heavy respectively, and p,q are complex mixing parameters

$$|p|^2 + |q|^2 = 1 \tag{2.2}$$

$$\frac{p}{q} = \left[\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M^*_{12} - \frac{i}{2}\Gamma^*_{12}}\right]^{1/2} \tag{2.3}$$

with the definitions

$$\begin{aligned} M &\equiv \frac{1}{2}(m_L + m_H), \Delta m \equiv m_H - m_L \\ \Gamma &\equiv \frac{1}{2}(\gamma_L + \gamma_H), \quad \Delta\gamma \equiv \gamma_L - \gamma_H > 0 \end{aligned} \tag{2.4}$$

A pure P^0 or $\bar{P}^0$ at t = 0 evolves in time as

$$\begin{aligned} |P^0_{phys}(t)> &= g_+(t)|P^0> + \frac{q}{p}g_-(t)|\bar{P}^0> \\ |\bar{P}^0_{phys}(t)> &= \frac{p}{q}g_-(t)|P^0> + g_+(t)|\bar{P}^0> \end{aligned} \tag{2.5}$$

where $g_\pm(t) = \frac{1}{2}(e^{-i\lambda_L t} \pm e^{-i\lambda_H t})$. If the neutral mesons have very short life-time, we have to consider the time-integrated effects. Denote the CP-conjugate state of the final state f by $\bar{f}$, $|\bar{f}>\equiv CP|f>$, our CP asymmetry is defined as

$$\mathcal{A}_{CP}(f) = \frac{\Gamma(P^0_{phys} \to f) - \Gamma(\bar{P}^0_{phys} \to \bar{f})}{\Gamma(P^0_{phys} \to f) + \Gamma(\bar{P}^0_{phys} \to \bar{f})} \tag{2.6}$$

where

$$\begin{aligned} \Gamma(P^0_{phys} \to f) &= \int_0^\infty dt |<f|\mathcal{H}|P^0_{phys}(t)>|^2 \\ \Gamma(\bar{P}^0_{phys} \to \bar{f}) &= \int_0^\infty dt |<f|\mathcal{H}|\bar{P}^0_{phys}(t)>|^2 \end{aligned} \tag{2.7}$$

From (2.5), we obtain the transition amplitude of a neutral meson decaying into the final state f as

$$\begin{aligned} <f|\mathcal{H}|P^0_{phys}(t)> &= g_+(t)A(f) + \frac{q}{p}g_-(t)\bar{A}(f) \\ <\bar{f}|\mathcal{H}|\bar{P}^0_{phys}(t)> &= \frac{p}{q}g_-(t)A(\bar{f}) + g_+(t)\bar{A}(\bar{f}) \end{aligned} \tag{2.8}$$

where $A(f)\equiv<f|\mathcal{H}|P^0>, \bar{A}(f)\equiv<f|\mathcal{H}|\bar{P}^0>, A(\bar{f})\equiv<\bar{f}|\mathcal{H}|P^0>, \bar{A}(\bar{f})\equiv<\bar{f}|\mathcal{H}|\bar{P}^0>$. For convenience, we define the ratio of these two amplitudes as

$$\rho_f \equiv \frac{\bar{A}(f)}{A(f)} \qquad \rho_{\bar{f}} \equiv \frac{\bar{A}(\bar{f})}{A(\bar{f})} \tag{2.9}$$

then we obtain

$$\begin{aligned} &\Gamma(P^0_{phys} \to f) \\ &= |A(f)|^2 \left[G_+ + G_-|\frac{q}{p}\rho_f|^2 \right. \\ &\left. + 2(ReG_{+-})Re\left(\frac{q}{p}\rho_f\right) - 2\left(ImG_{+-}\right)Im\left(\frac{q}{p}\rho_f\right)\right] \end{aligned} \tag{2.10}$$

$$\begin{aligned} &\Gamma\left(\bar{P}^0_{phys} \to \bar{f}\right) \\ &= |\bar{A}\left(\bar{f}\right)|^2 \left[G_+ + G_-|\frac{p}{q}\frac{1}{\rho_{\bar{f}}}|^2 \right. \\ &\left. + 2\left(ReG_{+-}\right)Re\left(\frac{p}{q}\frac{1}{\rho_{\bar{f}}}\right) - 2\left(ImG_{+-}\right)Im\left(\frac{p}{q}\frac{1}{\rho_{\bar{f}}}\right)\right] \end{aligned} \tag{2.11}$$

where

$$\begin{aligned} &G_+ \equiv \int_0^\infty dt|g_+(t)|^2, G_- \equiv \int_0^\infty dt|g_-(t)|^2, \\ &G_{+-} \equiv \int_0^\infty dt\ g^*_+(t)g_-(t). \end{aligned} \tag{2.12}$$

In the equations above, the x and y are two dimensionless mixing parameters, $x \equiv \frac{\Delta m}{\Gamma}$, $y \equiv \frac{\Delta\gamma}{2\Gamma}$. Note that both x and y are positive in our definition. From these preliminary formulas, we will discuss the time-independent CP asymmetry for neutral $P^0 - \bar{P}^0$ meson decays to CP eigenstates and non-CP eigenstates in Sects. 2.2 and 2.3 respectively.

2.2 Decays to CP eigenstates

If the neutral meson decays to CP eigenstates, i.e., $|\bar{f}>=\pm|f>$, so we have

$$|A(\bar{f})| = |A(f)|, \quad |\bar{A}(\bar{f})| = |\bar{A}(f)|, \quad \rho_{\bar{f}} = \rho_f \quad (2.13)$$

Put

$$\frac{q}{p} \equiv |\frac{q}{p}|e^{i\alpha}, \quad \rho_f \equiv |\rho_f|e^{i\beta_f}, \quad \frac{q}{p}\rho_f \equiv |\frac{q}{p}\rho_f|e^{i\phi_f} \quad (2.14)$$

then we have

$$\phi_f = \alpha + \beta_f \quad (2.15)$$

where ϕ_f is rephasing invariant [15]. It is easy to see that

$$\frac{p}{q}\rho_f^* = |\frac{p}{q}\rho_f|e^{-i\phi_f} \quad (2.16)$$

Substituting (2.14), (2.16), and (2.12) into (2.6), we find

$$\mathcal{A}_{cp}(f) = \frac{N_1}{D_1} \quad (2.17)$$

where

$$\begin{aligned} N_1 &= (2 + x^2 - y^2)(1 - |\rho_f|^2) \\ &\quad +(x^2 + y^2)[|\frac{q}{p}|^2|\rho_f|^2 - |\frac{p}{q}|^2] \\ &\quad -2(1 + x^2)y|\rho_f|[|\frac{q}{p}| - |\frac{p}{q}|]cos\phi_f \\ &\quad -2(1 - y^2)x|\rho_f|[|\frac{q}{p}| + |\frac{p}{q}]sin\phi_f \\ D_1 &= (2 + x^2 - y^2)(1 + |\rho_f|^2) \\ &\quad +(x^2 + y^2)[|\frac{q}{p}|^2|\rho_f|^2 + |\frac{p}{q}|^2] \\ &\quad -2(1 + x^2)y|\rho_f|[|\frac{q}{p}| + |\frac{p}{q}|]cos\phi_f \\ &\quad -2(1 - y^2)x|\rho_f|[|\frac{q}{p}| - |\frac{p}{q}|sin\phi_f. \quad (2.17) \end{aligned}$$

We now define the CP violation parameters ϵ and $\tilde{\epsilon}$ as

$$\epsilon \equiv \frac{1 - \frac{q}{p}}{1 + \frac{q}{p}}, \quad (2.18)$$

$$\tilde{\epsilon} \equiv \frac{1 - |\frac{q}{p}|^2}{1 + |\frac{q}{p}|^2} = |p|^2 - |q|^2 =< P_H^0|P_L^0 > \quad , \quad (2.19)$$

from (2.3),

$$\begin{aligned} \tilde{\epsilon} &= \frac{2Re\epsilon}{1 + |\epsilon|^2} \\ &= \frac{Im(M_{12}^*\Gamma_{12})}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + \frac{1}{4}(\Delta m)^2 + \frac{1}{16}(\Delta\gamma)^2} \quad (2.20) \end{aligned}$$

We know that ϵ depends on the phase convention, so it is not a physical quantity, but the $\tilde{\epsilon}$ is a real CP violation parameter which is rephasing invariant. Actually the $\tilde{\epsilon}$ denotes the CP violation in the amplitude modulus, i.e. $|\frac{q}{p}|^2 \neq 1$. If $|\epsilon|$ is of the order 10^{-3} from (2.20), $|\tilde{\epsilon}| \leq 2|\epsilon| \sim 10^{-3}$.

For CP eigenstates f, $|\rho_f| = \frac{|\bar{A}(f)|}{|A(f)|} = \frac{|\bar{A}(\bar{f})|}{|A(f)|}$. We define the direct CP violating asymmetry Δ_f as

$$\Delta_f = \frac{|A(f)|^2 - |\bar{A}(\bar{f})|^2}{|A(f)|^2 + |\bar{A}(\bar{f})|^2} = \frac{1 - |\rho_f|^2}{1 + |\rho_f|^2} \quad (2.21)$$

Assuming $|\Delta_f| \ll 1$ and also $x \ll 1$ (this is true for $D^0 - \bar{D}^0$ system), $y \ll 1$, then we obtain a simple CP asymmetry formula

$$\mathcal{A}_{CP}(f) \approx \Delta_f - xsin\phi_f + y\tilde{\epsilon}cos\phi_f - (x^2 + y^2)\tilde{\epsilon}. \quad (2.22)$$

We can see that there are four CP violating terms in (2.22). The first term Δ_f measures the direct CP violation in the transition amplitudes of the neutral meson decays. The other terms measure the indirect CP violation: the second and the third terms arise from the interplay between mixing and decay, the fourth term arises from only the mixing which is independent of the particular decay modes. The magnitude of the indirect CP violation depends on the four dimensionless and rephasing-invariant mixing parameters $x, y, \tilde{\epsilon}$ and ϕ_f, in which only ϕ_f relates to the specific decay final state. We emplasize that sometimes the CP violating terms directly proportional to $\tilde{\epsilon}$ are also important in comparison with the other terms, although they are small and can be often neglected (i.e. taking approximation $|p|^2 = |q^2|$) in the early discussion for CP asymmetry.

To discuss the direct CP violating asymmetry Δ_f, we may, without loss of generality, write the decay transition amplitude as

$$A(f) = G_1\ T_1\ e^{i\delta_1} + G_2\ T_2\ e^{i\delta_2} \quad (2.23)$$

where G_1, G_2 both are multiplication of two CKM matrix elements, δ_1, δ_2 are the strong phases and T_1, T_2 denote the real (positive) decay amplitude. With the CPT invariance, the CP-conjugate amplitude is

$$\bar{A}(\bar{f}) = G_1^*\ T_1\ e^{i\delta_1} + G_2^*\ T_2\ e^{i\delta_2} \quad (2.24)$$

We can obtain the direct CP vialating asymmetry as

$$\Delta_f = \frac{2Im(G_1^*G_2)sin(\delta_1 - \delta_2)}{|G_1|^2T_1/T_2 + |G_2|^2T_2/T_1 + Re(G_1^*G_2)cos(\delta_1 - \delta_2)} \quad (2.25)$$

$$\mathcal{A}_{cp}(f) = \frac{|\rho_f| sin\phi_f^{(+)}[(ysin\phi_f^{(-)} - xcos\phi_f^{(-)}] + |\rho_f| cos\phi_f^{(+)}[ycos\phi_f^{(-)} + xsin\phi_f^{(-)}]\tilde{\epsilon} - |\rho_f|^2(x^2+y^2)\tilde{\epsilon}}{1 + \frac{1}{2}|\rho_f|^2(x^2+y^2) - |\rho_f| cos\phi_f^{(+)}[ycos\phi_f^{(-)} + xsin\phi_f^{(-)}]} \quad (2.34)$$

It is clear that the Δ_f will vanish in both the cases of $Im(G_1^* G_2) = 0$ or $sin(\delta_1 - \delta_2) = 0$. In the case of $Im(G_1^* G_2) = 0$, the decay amplitude must have only a single weak phase. If not, we have $G_1 \neq G_2$ and $|Im(G_1^* G_2)|$ is a non-zero constant [16]

$$|ImG_1^* G_2| = J = c_1 c_2 c_3 s_1^2 s_2 s_3 s_\delta \quad (2.26)$$

Since the difficulty both in theoretical calculation and experimental measurement, the strong phase-shift $(\delta_1 - \delta_2)$ is uncertain. We will use extermum method to detour over this hurdle. At the maximum point of the $\Delta_f[(\delta_1 - \delta_2)]$, $cos(\delta_1 - \delta_2) = \frac{-2Re(G_1^* G_2)}{|G_1|^2 T_1/T_2 + |G_2|^2 T_2/T_1}$, we obtain the upper bound of direct CP violating asymmetry

$$|\Delta_f| \leq \frac{2|Im(G_1^* G_2)|}{\sqrt{(|G_1|^2 T_1/T_2 + |G_2|^2 T_2/T_1)^2 - 4[Re(G_1^* G_2)]^2}}. \quad (2.27)$$

Using these formulas, we will discuss the direct CP violation for the decays of the $D^0 - \bar{D}^0$ system in Sect. 3.2 .

2.3 Decays to non-CP eigenstates

If both P^0 and $\bar{P}^0$ mesons can decay into a common non-CP eigenstates, we now only discuss the following cases in which the decay amplitudes can be factorized formally as

$$A(f) = G_f T_f \; e^{i\delta_f} \;, \qquad \bar{A}(f) = \bar{G}_f \bar{T}_f \; e^{i\bar{\delta}_f} \quad (2.28)$$

and

$$\bar{A}(\bar{f}) = G_f^* T_f \; e^{i\delta_f} \;, \qquad A(\bar{f}) = \bar{G}_f^* \bar{T}_f \; e^{i\bar{\delta}_f} \quad (2.29)$$

This means that the decay amplitudes have only a single weak phase or a single strong phase. For example, if only a single weak phase G_f is involved, then $A(f) = G_f(T_1 \; e^{i\delta_1} + T_2 \; e^{i\delta_2}) = G_f T_f \; e^{i\delta_f}$. Where T_f and δ_f are functions of $T_1, T_2, \delta_1, \delta_2$. In most cases of decays to non-CP eigenstates, the condition (2.28) is usually satisfied. Obviously there are no direct CP violation in these decay processes, so we have the relations

$$|\bar{A}(\bar{f})| = |A(f)|, \quad |A(\bar{f})| = |\bar{A}(f)| \quad |\rho_{\bar{f}}| = \frac{1}{|\rho_f|} \quad (2.30)$$

From (2.10) and (2.11) and the conditions (2.30). We obtain

$$\mathcal{A}_{cp}(f) = \frac{N_2}{D_2} \quad (2.31)$$

where

$$\begin{aligned} N_2 &= G_-|\rho_f|^2[|\frac{q}{p}|^2 - |\frac{p}{q}|^2] + 2Re(G_{+-})[|\frac{q}{p}|cos\phi_f \\ &\quad -|\frac{p}{q}|cos\phi_{\bar{f}}] - 2Im(G_{+-})|\rho_f|[|\frac{q}{p}|sin\phi_f + |\frac{p}{q}|sin\phi_{\bar{f}}] \\ D_2 &= 2G_+ + G_-|\rho_f|^2[|\frac{q}{p}|^2 + |\frac{p}{q}|^2] \\ &\quad +2Re(G_{+-})|\rho_f|[|\frac{q}{p}|cos\phi_f + |\frac{p}{q}|cos\phi_{\bar{f}}] \\ &\quad -2Im(G_{+-})|\rho_f|[|\frac{q}{p}|sin\phi_f - |\frac{p}{q}|sin\phi_{\bar{f}}]. \end{aligned}$$

where the $\phi_f, \phi_{\bar{f}}$ are rephasing-invariant and are defined as

$$\frac{q}{p}\rho_f \equiv |\frac{q}{p}\rho_f| e^{i\phi_f} \;, \frac{q}{p}\rho_{\bar{f}} \equiv |\frac{q}{p}\rho_{\bar{f}}| e^{i\phi_{\bar{f}}} \quad (2.32)$$

After some arithmetics, we obtain the CP asymmetry formula as follows

$$\mathcal{A}_{cp}(f) \approx \frac{N_3}{D_3}, \quad (2.33)$$

where

$$\begin{aligned} N_3 &= |\rho_f| sin\phi_f^{(+)}[(1+x^2)ysin\phi_f^{(-)} - (1-y^2)xcos\phi_f^{(-)}] \\ &\quad +|\rho_f| cos\phi_f^{(+)}[(1+x^2)ycos\phi_f^{(-)} \\ &\quad +(1-y^2)xsin\phi_f^{(-)}]\tilde{\epsilon} - |\rho_f|^2(x^2+y^2)\tilde{\epsilon} \\ D_3 &= \frac{1}{2}(2+x^2-y^2) + \frac{1}{2}|\rho_f|^2(x^2+y^2) \\ &\quad -|\rho_f| cos\phi_f^{(+)}[(1+x^2)ycos\phi_f^{(-)} + (1-y^2)xsin\phi_f^{(-)}] \end{aligned}$$

where $\phi_f^{(\pm)} \equiv \frac{1}{2}(\phi_f \pm \phi_{\bar{f}})$.

If $x^2 \ll 1, y^2 \ll 1$,then the CP asymmetry formula (2.33) can be simplified as (see (2.34) on top of the page). We can see that there is a factor $|\rho_f|$ which will enhance $(|\rho_f| > 1)$ or suppress $(|\rho_f| < 1)$ the CP asymmetry. From (2.28), (2.29) $\rho_f, \rho_{\bar{f}}$ are given as

$$\begin{aligned} \rho_f &\equiv |\rho_f| e^{i\beta_f} \equiv \frac{\bar{A}(f)}{A(f)} = \frac{\bar{G}_f \bar{T}_f e^{i\bar{\delta}_f}}{G_f T_f e^{i\delta_f}} \\ \rho_{\bar{f}} &\equiv |\rho_{\bar{f}}| e^{i\beta_{\bar{f}}} \equiv \frac{\bar{A}(\bar{f})}{A(\bar{f})} = \frac{G_f^* T_f e^{i\delta_f}}{\bar{G}_f^* \bar{T}_f e^{i\bar{\delta}_f}} \end{aligned} \quad (2.35)$$

Define the weak phases as $G_f \equiv |G_f| e^{i\theta_f}$, $\bar{G}_f \equiv |\bar{G}_f| e^{i\bar{\theta}_f}$, we have $\beta_f = (\bar{\theta}_f - \theta_f) + (\bar{\delta}_f - \delta_f)$ and $\beta_{\bar{f}} = (\bar{\theta}_f - \theta_f) - (\bar{\delta}_f - \delta_f)$. Then we find that the $\phi_f^{(+)}$ is independent of the strong phases and the $\phi_f^{(-)}$ is related to the strong phase only:

$$\phi_f^{(+)} = \alpha + (\bar{\theta}_f - \theta_f) \qquad \phi_f^{(-)} = \bar{\delta}_f - \delta_f \quad (2.36)$$

$$\mathcal{A}_{CP}^{(-)}(f_1f_2) \approx \frac{2(1-|\rho_{f_1}|^2|\rho_{f_2}|^2)(x^2+y^2)\tilde{\epsilon}+4|\rho_{f_1}||\rho_{f_2}|sin(\phi_{f_1}^{(+)}-\phi_{f_2}^{(+)})sin(\phi_{f_1}^{(-)}-\phi_{f_2}^{(-)})}{(1+|\rho_{f_1}|^2|\rho_{f_2}|^2)(x^2+y^2)-4|\rho_{f_1}||\rho_{f_2}|cos(\phi_{f_1}^{(+)}-\phi_{f_2}^{(+)})cos(\phi_{f_1}^{(-)}-\phi_{f_2}^{(-)})} \quad (2.43)$$

(2.34) and (2.36) are the master formulas for our discussion concerning the $D^0-\bar{D}^0$ decays to non-CP eigenstates.

2.4 Coherent decays of neutral meson pairs

In most cases the $P^0\bar{P}^0$ are produced in pair. Before decay the coherent $P^0\bar{P}^0$ pair is described by the wavefunction

$$|i(t,t_2)>_\eta = \frac{1}{\sqrt{2}}[|p^0_{phys}(t_1)>_L |\bar{P}^0_{phys}(t_2)>_R + \eta|P^0_{phys}(t_2)>_R |\bar{P}^0_{phys}(t_1)>_L] \quad (2.37)$$

where L and R signify the three-momentum vecter of the neutral mesons, and $\eta=\pm1$ denotes the charge-conjugation parity of this coherent system. We consider a joint decay process that one of the two neutral mesons (denoted by L) decays into a final state f_1 at proper time t_1 and the other (denoted by R) decays into f_2 at t_2. After a lengthy calculation, we find

$$\begin{aligned}&< f_1t_1; f_2t_2|\mathcal{H}|i(t_1,t_2)>_\eta \\ &= \frac{1}{\sqrt{2}}A(f_1)A(f_2)\{(\frac{p}{q}+\eta\frac{q}{p}\rho_{f_1}\rho_{f_2})[g_+(t_1)g_-(t_2) \\ &+\eta g_-(t_1)g_+(t_2)]+(\rho_{f_1}+\eta\rho_{f_2})[g_+(t_1)g_+(t_2) \\ &+\eta g_-(t_1)g_+(t_2)]\}\end{aligned} \quad (2.38)$$

$$\Gamma(f_1,f_2)_\eta \equiv \int_0^\infty\int_0^\infty dt_1dt_2|< f_1t_1; f_2t_2|\mathcal{H}i(t_1t_2)>_\eta| \quad (2.39)$$

We can get expressions for $\Gamma(\bar{f}_1,\bar{f}_2)_\eta$ and $\Gamma(f_1,\bar{f}_2)_\eta$ by replacement $f_1\to\bar{f}_1$, $f_2\to\bar{f}_2$ in (2.39) where $\bar{f}_1$ and $\bar{f}_2$ are CP-conjugate states of f_1 and f_2 respectively

$$\begin{aligned}\Gamma(\bar{f}_1,\bar{f}_2)_\eta &\equiv \int_0^\infty\int_0^\infty dt_1dt_2|<\bar{f}_1t_1;\bar{f}_2t_2|\mathcal{H}|i(t_1t_2)>_\eta|^2 \\ \Gamma(f_1,\bar{f}_2)_\eta &\equiv \int_0^\infty\int_0^\infty dt_1dt_2|< f_1t_1;\bar{f}_2t_2|\mathcal{H}|i(t_1t_2)>_\eta|^2\end{aligned} \quad (2.40)$$

For simplicity we will discuss the case of $\eta=-1$ only (the case of $\eta=+1$ can be discussed with the same method).

(i) f_1 and f_2 both are non-CP eigenstates

Similar as in Sect. 2.3 assuming that there is no direct CP violation in the case of decays to non-CP eigenstates, it means $|A(\bar{f}_i)|=|\bar{A}(f_i)|$, $|\bar{A}(\bar{f}_i)|=|A(f_i)|$ and $|\rho_{\bar{f}_i}|=\frac{1}{|\rho_{f_i}|}(i=1,2)$, then we find

$$\begin{aligned}\mathcal{A}_{CP}^{(-)}(f_1,f_2) &\equiv \frac{\Gamma(f_1,f_2)_- - \Gamma(\bar{f}_1,\bar{f}_2)_-}{\Gamma(f_1,f_2)_- + \Gamma(\bar{f}_1,\bar{f}_2)_-} = \frac{\Gamma_-}{\Gamma_+} \\ \Gamma_- &= (1-|\rho_{f_1}|^2|\rho_{f_2}|^2)(|\frac{p}{q}|^2-|\frac{q}{p}|^2)(x^2+y^2) \\ &-4|\rho_{f_1}||\rho_{f_2}| \\ &\times[(1+x^2)(cos\phi_{f_1}cos\phi_{f_2}-cos\phi_{\bar{f}_1}cos\phi_{\bar{f}_2}) \\ &+(1-y^2)(sin\phi_{f_1}sin\phi_{f_2}-sin\phi_{\bar{f}_1}sin\phi_{\bar{f}_2})] \\ \Gamma_+ &= (1+|\rho_{f_1}|^2|\rho_{f_2}|^2)(|\frac{p}{q}|^2+|\frac{q}{p}|^2)(x^2+y^2) \\ &-4|\rho_{f_1}||\rho_{f_2}|[(1+x^2)(cos\phi_{f_1}cos\phi_{f_2} \\ &+cos\phi_{\bar{f}_1}cos\phi_{\bar{f}_2})+(1-y^2)(sin\phi_{f_1}sin\phi_{f_2} \\ &+sin\phi_{\bar{f}_1}sin\phi_{\bar{f}_2})]\end{aligned} \quad (2.41)$$

For the particular case $\rho_{f_2}=0$ (or $\rho_{f_1}=0$), we find

$$\mathcal{A}_{CP}^{(-)}(f_1,f_2) = \frac{|\frac{p}{q}|^2-|\frac{q}{p}|^2}{|\frac{p}{q}|^2+|\frac{q}{p}|^2} = \frac{2\tilde{\epsilon}}{1+\tilde{\epsilon}^2} \approx 2\tilde{\epsilon} \quad (2.42)$$

In that case the CP violation is independent of the final-state f_1 (or f_2). Therefore the formula (2.42) is very useful to measure the mixing parameter $\tilde{\epsilon}$ experimentally.

If $x^2 \ll 1$, $y^2 \ll 1$, we can further simplify the CP asymmetry (2.41) as follows (see (2.43) on top of the page). where $\phi_{f_i}^{\pm} \equiv \frac{1}{2}(\phi_{f_i}\pm\phi_{\bar{f}_i})$ $(i=1,2)$.

(ii) f_1 is CP eigenstates and f_2 is non-CP eigenstates

Assuming that there is no direct CP violation for decays to non-CP eigenstates, i.e. $|A(\bar{f}_2)|=|\bar{A}(f_2)|$, $|\bar{A}(\bar{f}_2)|=|A(f_2)|$, and $|\rho_{\bar{f}_2}|=\frac{1}{|\rho_{f_2}|}$, then we have

$$\begin{aligned}\mathcal{A}_{CP}^{(-)}(f_1f_2) &\equiv \frac{\Gamma(f_1,f_2)_- - \Gamma(\bar{f}_1,\bar{f}_2)_-}{\Gamma(f_1,f_2)_- + \Gamma(\bar{f}_1,\bar{f}_2)_-} \\ &= \frac{\Gamma(f_1,f_2)_- - \Gamma(f_1,\bar{f}_2)_-}{\Gamma(f_1,f_2)_- + \Gamma(f_1,\bar{f}_2)_-} = \frac{\Gamma'_-}{\Gamma'_+} \\ \Gamma'_- &= (1-|\rho_{f_2}|^2)[(|\frac{p}{q}|^2-|\frac{q}{p}|^2|\rho_{f_1}|^2)(x^2+y^2) \\ &+(|\rho_{f_1}|^2-1)(2+x^2-y^2)] \\ &-4|\rho_{f_1}||\rho_{f_2}|[(1+x^2) \\ &\times cos\phi_{f_1}(cos\phi_{f_2}-cos\phi_{\bar{f}_2}) \\ &+(1-y^2)sin\phi_{f_1}(sin\phi_{f_2}-sin\phi_{\bar{f}_2})] \\ \Gamma'_+ &= (1+|\rho_{f_2}|^2)[(|\frac{p}{q}|^2+|\frac{q}{p}|^2|\rho_{f_1}|^2)(x^2+y^2) \\ &+(|\rho_{f_1}|^2+1)(2+x^2-y^2)] \\ &+4|\rho_{f_1}||\rho_{f_2}|[(1+x^2) \\ &\times cos\phi_{f_1}(cos\phi_{f_2}+cos\phi_{\bar{f}_2}) \\ &+(1-y^2)sin\phi_{f_1}(sin\phi_{f_2}+sin\phi_{\bar{f}_2})]\end{aligned} \quad (2.44)$$

$$\mathcal{A}_{CP}^{(-)}(f_1,f_2)\approx\frac{(1-|\rho_{f_2}|^2)[-\Delta_{f_1}+(x^2+y^2)\tilde{\epsilon}]-2|\rho_{f_2}|sin(\phi_{f_1}-\phi_{f_2}^{(+)})sin\phi_{f_2}^{(-)}}{1+|\rho_{f_2}|^2-2|\rho_{f_2}|cos(\phi_{f_1}-\phi_{f_2}^{(+)})cos\phi_{f_2}^{(-)}} \tag{2.45}$$

$$\begin{aligned}\mathcal{A}_{CP}^{(f)}&=\frac{\Gamma(D^0_{phys}\to f)-\Gamma(\bar{D}^0_{phys}\to \bar{f})}{\Gamma(D^0_{phys}\to f)+\Gamma(\bar{D}^0_{phys}\to \bar{f})}\\&\approx\frac{|\rho_f|sin\phi_f^{(+)}[ysin\phi_f^{(-)}-xcos\phi_f^{(-)}]+|\rho_f|cos\phi_f^{(+)}[ycos\phi_f^{(-)}+xsin\phi_f^{(-)}]\tilde{\epsilon}-|\rho_f|^2(x^2+y^2)\tilde{\epsilon}}{1+\frac{1}{2}|\rho_f|^2(x^2+y^2)-|\rho_f|cos\phi_f^{(+)}[ycos\phi_f^{(-)}+xsin\phi_f^{(-)}]}\end{aligned} \tag{3.3}$$

if $x^2\ll 1,\ y^2\ll 1$, and using the relation $|\frac{q}{p}|^2=\frac{1-\tilde{\epsilon}}{1+\tilde{\epsilon}}$, $|\rho_{f_1}|^2=\frac{1-\Delta_{f_1}}{1+\Delta_{f_1}}$, the CP asymmetry (2.44) reads (see (2.45) on top of the page). Since $\phi_{\bar{f}_2}^{(+)}=\phi_{f_2}^{(+)}$, $\phi_{\bar{f}_2}^{(-)}=-\phi_{f_2}^{(-)}$ and $|\rho_{\bar{f}_2}|=\frac{1}{|\rho_{f_2}|}$, it is seen that the approximate CP asymmetry formula (2.45) still satisfy the relation $\mathcal{A}_{CP}^{(-)}(f_1,\bar{f}_2)=-\mathcal{A}_{CP}^{(-)}(f_1,f_2)$ for the case of the f_1 being CP eigenstates. If $|\rho_{f_2}|\ll 1$, we obtain the simple CP asymmetry formula

$$\begin{aligned}\mathcal{A}_{CP}^{(-)}(f_1,f_2)=&-\Delta_{f_1}+(x^2+y^2)\tilde{\epsilon}\\&-2|\rho_{f_2}|sin(\phi_{f_1}-\phi_{f_1}^{(+)})sin\phi_{f_2}^{(-)}\end{aligned} \tag{2.46}$$

for the particular case $\rho_{f_2}=0$, we get

$$\mathcal{A}_{CP}^{(-)}(f_1,f_2)=-\Delta_{f_1}+(x^2+y^2)\tilde{\epsilon} \tag{2.47}$$

in comparision with (2.22), we obtain the relation for the case of $\rho_{f_2}=0$

$$\mathcal{A}_{CP}(f_1)+\mathcal{A}_{CP}^{(-)}(f_1,f_2)=-xsin\phi_{f_1}+y\tilde{\epsilon}cos\phi_{f_1} \tag{2.48}$$

The formulas (2.47) and (2.48) provide a method to separate the direct CP violation and the indirect CP violation arising from the interplay between mixing and decay for the case of decays to CP eigenstate f_1. It is very useful specially, for the decay process with semileptonic tagging.

3 CP violation in neutral D meson decays

Utilizing the CP asymmetry formulas derived in the sections above, we will discuss systematically the $D^0-\bar{D}^0$ mixing and CP violation for neutral D meson decays to a variety of CP eigenstates and non-CP eigenstates in this section. We analyse in detail the direct and indirect CP violation within the standard model and obtain the upper bound of the direct and indirect CP violation in neutral D meson decays. These results are very useful for searching CP-violating singnals in neutral D meson decays, for measuring the mixing parameters of the $D^0-\bar{D}^0$ system and for searching new physics beyond the standard model.

3.1 CP asymmetries for the decays of the $D^0-\bar{D}^0$ system

In experiment, the incoherent single $D^0(\bar{D}^0)$ can be obtained in the decay process $\Psi(4.14)\to D^-D^{*+}\to\pi^+D^-D^0$ and its conjugate decay $\Psi(4.14)\to D^+D^{*-}\to\pi^-D^+D^0$, while the coherent $D^0\bar{D}^0$ pair can be obtained in the decay processes $\Psi(3.77)\to D^0\bar{D}^0$ or $\Psi(4.14)\to D^{*0}\bar{D}^0\to\pi^0D^0\bar{D}^0$. There are three dimensionless mixing parameters x_D,y_D and $\tilde{\epsilon}_D$ which are related to the $D^0-\bar{D}^0$ system only,

$$\begin{aligned}x_D&\equiv\frac{\Delta m_D}{\Gamma_D}=\frac{m_H^{(D)}-m_L^{(D)}}{1/2(\gamma_L^{(D)}+\gamma_H^{(D)})}>0,\\y_D&\equiv\frac{\Delta\Gamma_D}{2\Gamma_D}=\frac{\gamma_L^{(D)}-\gamma_H^{(D)}}{\gamma_L^{(D)}+\gamma_H^{(D)}}>0,\\\tilde{\epsilon}_D&\equiv|p_D|^2-|q_D|^2\end{aligned} \tag{3.1}$$

At present the experimental upper bounds for $D^0-\bar{D}^0$ mixing are $r_D\approx\frac{1}{2}(x^2+y^2)<4.7\times10^{-3}$ (FNAL E791) and $r_D\approx\frac{1}{2}(x^2+y^2)<3.7\times10^{-3}$ (FNAL E691). It is also believed that $|\tilde{\epsilon}_D|\sim10^{-3}$, so $x_D\ll1$, $y_D\ll1$ and $|\tilde{\epsilon}_D|\ll1$. For convenience, we will omit the subscript "D" below. According to (2.22) and (2.34), the CP asymmetries for the incoherent $D^0(\bar{D}^0)$ decays to the CP eigenstates and the non-CP eigenstates are

$$\begin{aligned}\mathcal{A}_{cp}(f=\pm\bar{f})&=\frac{\Gamma(D^0_{phys}\to f)-\Gamma(\bar{D}^0_{phys}\to f)}{\Gamma(D^0_{phys}\to f)+\Gamma(\bar{D}^0_{phys}\to f)}\\&\approx\Delta_f-xsin\phi_f+y\tilde{\epsilon}cos\phi_f-(x^2+y^2)\tilde{\epsilon}\end{aligned} \tag{3.2}$$

(see (3.3) on top of the page) where $\phi_f^{(\pm)}\equiv\frac{1}{2}(\phi_f\pm\phi_{\bar{f}})$ For the case of the coherent $D^0\bar{D}^0$ pair decay, from (2.43) and (2.45), we have (see (3.4) and (3.5) on top of the next page). There is an important simplification for the decays with semileptonic tagging. Assuming the the semileptonic final-state is $|f_2>=|\ell^+X^->$, since the process $\bar{D}^0\to\ell^+X^-$ is forbidden according to the $\Delta Q=\Delta C$ rule, $\bar{A}(f_2)=<\ell^+X^-|\mathcal{H}|\bar{D}^0>=0$, $|\rho_{f_2}|=\frac{|\bar{A}(f_2)|}{|A(f_2)|}=0$, from (3.4) and (3.5), we get

$$\mathcal{A}_{CP}^{(-)}(f,\ell^+X^-)=2\tilde{\epsilon} \tag{3.6}$$

$$\mathcal{A}_{CP}^{(-)}(f=\pm\bar{f},\ell^+X^-)=-\Delta_f+(x^2+y^2)\tilde{\epsilon} \tag{3.7}$$

Because $\mathcal{A}_{CP}^{(-)}(\bar{f}_1,\bar{f}_2)=-\mathcal{A}_{CP}^{(-)}(f_1,f_2)$, we have similarly $\mathcal{A}_{CP}^{-}(f,\ell^-X^+)\approx-2\tilde{\epsilon}$, $\mathcal{A}_{CP}^{(-)}(f=\pm\bar{f},\ell^-X^+)\approx\Delta_f-(x^2+$

$$\mathcal{A}_{CP}^{(-)}(f_1, f_2) = \frac{\Gamma(f_1, f_2)_- - \Gamma(\bar{f}_1, \bar{f}_2)_-}{\Gamma(f_1, f_2)_- + \Gamma(\bar{f}_1, \bar{f}_2)_-}$$
$$\approx \frac{2(1-|\rho_{f_1}|^2|\rho_{f_2}|^2)(x^2+y^2)\tilde{\epsilon} + 4|\rho_{f_1}||\rho_{f_2}|sin(\phi_{f_1}^{(+)} - \phi_{f_2}^{(+)})sin(\phi_{f_1}^{(-)} - \phi_{f_2}^{(-)})}{(1+|\rho_{f_1}|^2|\rho_{f_2}|^2)(x^2+y^2) - 4|\rho_{f_1}||\rho_{f_2}|cos(\phi_{f_1}^{(+)} - \phi_{f_2}^{(+)})cos(\phi_{f_1}^{(-)} - \phi_{f_2}^{(-)})} \quad (3.4)$$

$$\mathcal{A}_{CP}^{(-)}(f_1 = \pm\bar{f}_1, f_2) = \frac{\Gamma(f_1, f_2)_- - \Gamma(f_1, \bar{f}_2)_-}{\Gamma(f_1, f_2)_- + \Gamma(f_1, \bar{f}_2)_-} \approx \frac{(1-|\rho_{f_2}|^2)[-\Delta_{f_1} + (x^2+y^2)\tilde{\epsilon}] - 2|\rho_{f_2}|sin\phi_{f_1} - \phi_{f_2}^{(+)})sin(\phi_{f_2}^{(-)}}{1+|\rho_{f_2}|^2 - 2|\rho_{f_2}|cos(\phi_{f_1} - \phi_{f_2}^{(+)})cos\phi_{f_2}^{(-)}} \quad (3.5)$$

$y^2)\tilde{\epsilon}$. From (3.2) and (3.7), we find

$$\mathcal{A}_{CP}(f = \pm\bar{f}) + \mathcal{A}_{CP}^{(-)}(f = \pm\bar{f}, \ell^+ X^-) = -xsin\phi_f + y\tilde{\epsilon}cos\phi_f \quad (3.8)$$

(3.6), (3.7) and (3.8) are very useful formulas in experiment for measuring the mixing parameter $\tilde{\epsilon}$, the direct CP violation asymmetry Δ_f and the indirect CP violation of the neutral D mesons decays to the CP eigenstates respectively. Because the $\mathcal{A}_{CP}^{(-)}(f, \ell^\pm X^\pm)$ is independent of the decay final-state f, it is not only possible to increase statistics, but also possible to check the $\Delta Q = \Delta C$ rule of the standard model.

From our CP asymmetry formulas above, the magnitude of the CP violation depends on not only the mixing parameters $x, y, \tilde{\epsilon}$, but also the magnitude of the Δ_f, ϕ_f (for decays to CP eigenstales) and the $|\rho_f|, \phi_f^{(\pm)}$ (for decays to non-CP eigenstates) which all relate to the specific final-state of decay. In order to find the main effects of the CP violation in our CP asymmetry formulas, it is necessary and important to estimate these CP violation parameters. We will discuss these CP violation parameters for two-body decay processes in the next section.

3.2 Direct CP violation asymmetry Δ_f

The decay amplitude for the D^0 decays to CP eigenstates can be written as

$$A(f) = G_1 T_1 e^{i\delta_1} + G_2 T_2 e^{i\delta_2} \quad (3.9)$$

where the CKM elements are $G_1 = V_{ud}V_{cd}^*, G_2 = V_{us}V_{cs}^*$. According to the Wolfenstein representation of the CKM matrix [17]

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (3.10)$$
$$= \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

and taking the Wolfenstein parameter as [18] $\lambda = 0.22$, $A = 0.823$, $\eta = 0.336$, $\rho = 0.160$, we find

$$|G_1|^2 \approx |G_2|^2 \approx -(ReG_1^* G_2) \approx \lambda^2(1-\frac{\lambda^2}{2})^2 \quad (3.11)$$
$$J = ImG_1G_2^* = Im(V_{ud}V_{cs}V_{us}^*V_{cd}^*) \approx A^2\lambda^6(1-\frac{\lambda^2}{2})\eta \approx 2.52 \times 10^{-5} \quad (3.12)$$

from (2.25), our direct CP violation asymmetry is

$$\Delta_f \approx -\frac{2A^2\lambda^4\eta}{(1-\frac{\lambda^2}{2})} \cdot \frac{sin(\delta_1 - \delta_2)}{\frac{T_1}{T_2} + \frac{T_2}{T_1} - 2cos(\delta_1 - \delta_2)} \approx -1.09 \times 10^{-3} \frac{sin\delta_f}{h_f + \frac{1}{h_f} - 2cos\delta_f} \quad (3.13)$$

where $h_f \equiv \frac{T_1}{T_2} > 0$, $\delta_f \equiv \delta_1 - \delta_2$.

If $h_f >> 1$ or $h_f \ll 1$, in both cases the direct CP violation asymmetry will be very small

$$\Delta_f = -1.09 \times 10^{-3} \times \begin{cases} h_f sin\delta_f \ (h_f \ll 1) \\ \frac{1}{h_f} sin\delta_f \ (h_f >> 1) \end{cases} \quad (3.14)$$

For the decay of $D^0 \to K^+K^-$, since T_1 only contains the contribution of the penguin-diagram, it is much smaller comparing with T_2 which is dominanted by the tree-diagram and $h_f \sim 10^{-2} \ll 1$ [19]. For similar reason, for the decay of $D^0 \to \pi^+\pi^-, \pi^0\pi^0, \pi^0\rho^0, h_f \sim 10^2 >> 1$. From (3.13), we get

$$\Delta_f \approx -1.09 \times 10^{-5} sin\delta_f, \quad (3.15)$$
$$\text{so } |\Delta_f| < 1.09 \times 10^{-5} \quad (f = K^+K^-, \pi^+\pi^-, \pi^0\pi^0, \pi^0\rho^0)$$

For the decay of $D^0 \to \pi^0\eta$, $\pi^0\phi, \eta\phi, \eta\eta$, T_1 and T_2 are approximately equal in order of magnitude but $T_1 \neq T_2$ i.e. $h_f \sim o(1)$, from (2.27) and (3.11), (3.12), the upper bound of direct CP violation asymmetry is

$$|\Delta_f| \le \frac{3.25\eta \times 10^{-3}}{\sqrt{(T_1/T_2 + T_2/T_1)^2 - 4}} = \frac{3.25\eta \times 10^{-3}}{|h_f - 1/h_f|} = 1.09 \times 10^{-3} F(h_f) \quad (3.16)$$

where we take $\eta \approx 0.34$ and $F(h_f) = \frac{1}{|h_f - 1/h_f|} = \frac{h_f}{|h_f^2 - 1|}$. Obviously $F(h_f) = F(1/h_f)$ holds. Because $F(h_f)$ is an increasing function for $h_f < 1$, but a decreasing function for $h_f > 1$, then for $h_f < 4/5 = 0.8$ or $h_f > 5/4 = 1.25$, we get $F(h_f) < F(5/4) = 20/9 \approx 2.2$ and $|\Delta_f| < 2.40\times10^{-3}$; if $h_f < 1/5 = 0.2$ or $h_f > 5$, we get $F(h_f) < F(5) = 5/24 \approx 0.2$ and $|\Delta_f| < 2.18\times10^{-4}$.

Assuming $0.2 < h_f < 0.8$ or $1.25 < h_f < 5$ for the decay of $D^0 \to \pi^0\eta, \pi^0\phi, \eta\phi, \eta\eta$, we find the upper bound of direct CP violation asymmetry is

$$|\Delta_f| < 2\times(10^{-4} \sim 10^{-3}) \quad (f = \pi^0\eta, \quad \pi^0\phi, \ \ \eta\phi, \quad \eta\eta) \tag{3.17}$$

For the decay of $D^0 \to K^0\bar{K}^0, \phi\phi$, $T_1 = T_2$ (or $T_1 \approx T_2$), we must be careful to use the approximate Wolfenstein parameterization in this case. By the unitarity of the CKM matrix, $V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$, we find

$$|G_1|^2 + |G_2|^2 + 2(ReG_1^*G_2) = |V_{ub}V_{cb}^*|^2 \approx A^4\lambda^{10}(\rho^2+\eta^2) \tag{3.18}$$

For the case of $T_1 = T_2$, the direct CP violation formula (2.25) becomes

$$\begin{aligned}\Delta_f &= \frac{2Im(G_1^*G_2)sin(\delta_1-\delta_2)}{|G_1|^2+|G_2|^2+2Re(G_1^*G_2)cos(\delta_1-\delta_2)} \\ &= \frac{2Im(G_1^*G_2)sin\delta_f}{|V_{ub}V_{cb}^*|^2 - 2Re(G_1^*G_2)(1-cos\delta_f)} \\ &\approx -1.09\times10^{-3}\frac{sin\delta_f}{3.66\times10^{-7}+4sin^2(1/2\delta_f)}\end{aligned} \tag{3.19}$$

On the other hand, from (2.27), the upper bound for the direct CP violation asymmetry is

$$\begin{aligned}|\Delta_f| &\le \frac{2|Im(G_1^*G_2)|}{\sqrt{(|G_1|^2+|G_2|^2)^2 - 4[Re(G_1^*G_2)^2]}} \\ &\approx \frac{\eta}{\sqrt{\rho^2+\eta^2}} \approx 0.90 \quad (f = K^0\bar{K}^0, \quad \phi\phi)\end{aligned} \tag{3.20}$$

From (3.19) and (3.20), it can be seen that Δ_f is very sensitive to the strong phase-shift δ_f, In the very small region $0 \le sin\delta_f < 10^{-3}$, $|\Delta_f|$ can change from zero to about one. so if $sin\delta_f \ne 0$, the direct CP violation asymmetry could be quit large in the decay processes $D^0 \to K^0\bar{K}^0, \ \ \phi\phi$.

For the decay of $D^0 \to K_{1,2}\pi^0$, where the K_1 and K_2 both are the CP eigenstates with positive and negative CP parity respectively

$$\begin{aligned}&|K_{1.2}> = \frac{1}{\sqrt{2}}(|K^0> \pm|\bar{K}^0>, \\ &CP|K^0> = |\bar{K}^0>\end{aligned} \tag{3.21}$$

The decay amplitudes of $D^0 \to K_{1,2}\pi^0$ are then given by

$$\begin{aligned}A(K_{1,2}\pi^0) &= \frac{1}{\sqrt{2}}[A(K^0\pi^0) \pm A(\bar{K}^0\pi^0)] \\ \bar{A}(K_{1,2}\pi^0) &= \frac{1}{\sqrt{2}}[\bar{A}(K^0\pi^0) \pm \bar{A}(\bar{K}^0\pi^0)]\end{aligned} \tag{3.22}$$

Here the transition amplitudes of the decay $D^0(\bar{D}^0) \to K^0\pi^0$ can be factorized as

$$\begin{aligned}A(K^0\pi^0) &= V_{us}V_{cd}^*T_1e^{i\delta_1}, \\ A(\bar{K}^0\pi^0) &= V_{ud}V_{cs}^*T_2e^{i\delta_2} \\ \bar{A}(\bar{K}^0\pi^0) &= V_{us}^*V_{cd}T_1e^{i\delta_1} \\ \bar{A}(K^0\pi^0) &= V_{ud}^*V_{cs}T_2e^{i\delta_2}\end{aligned} \tag{3.23}$$

Denoting $G_1 = V_{us}V_{cd}^*$, $G_2 = V_{ud}V_{cs}^*$, we get

$$\begin{aligned}A(K_1\pi^0) &= \frac{1}{\sqrt{2}}(G_1T_1e^{i\delta_1} + G_2T_2e^{i\delta_2}) \\ A(K_2\pi^0) &= \frac{1}{\sqrt{2}}(G_1T_1e^{i\delta_1} + G_2T_2e^{i(\delta_2+\pi)})\end{aligned} \tag{3.24}$$

Using $|G_1| = \lambda^4, |G_2| = (1-\frac{\lambda_2}{2})^4$, $Re(G_1^*G_2) = -\lambda^2(1-\frac{\lambda_2}{2})^2$, $Im(G_1^*G_2) = -J = -A^2\lambda^6(1-\frac{\lambda^2}{2})\eta$, and assuming $h \equiv \frac{T_1}{T_2} = 1$ [20], from (2.25), we obtain

$$\begin{aligned}\Delta_{K_1\pi^0} &\approx 2(ImG_1^*G_2)sin(\delta_1-\delta_2) \\ &\approx -2Jsin(\delta_1-\delta_2)\end{aligned} \tag{3.25}$$

$$\begin{aligned}\Delta_{K_2\pi^0} &= \frac{-2Im(G_1^*G_2)sin(\delta_1-\delta_2)}{|G_1|^2h + |G_2|^2\frac{1}{h} - 2Re(G_1^*G_2)cos(\delta_1-\delta_2)} \\ &\approx -\Delta_{K_1\pi^0}\end{aligned} \tag{3.26}$$

$$|\Delta_{K_1\pi^0}| \approx |\Delta_{K_2\pi^0}| < 2J \approx 5\times10^{-5} \tag{3.27}$$

For the physical decay processes $D^0 \to K_{S,L}\pi^0$, the mass eigenstates K_s and K_L can be written as

$$\begin{aligned}|K_S> &= p_K|K^0> + q_K|\bar{K}^0> \\ |K_L> &= p_K|K^0> - q_K|\bar{K}^0>\end{aligned} \tag{3.28}$$

Since $p_K \ne q_K$, exactly speaking, both $K_S\pi^0$ and $K_L\pi^0$ are not CP eigenstates. By denoting $|\bar{K}_{S,L}> = CP|K_{S,L}> = p_K|\bar{K}^0> \pm q_K|K^0>$ and from (3.23), we have

$$\begin{aligned}A(K_{S,L}\pi^0) &= <K_{S,L}\pi^0|\mathcal{H}|D^0> \\ &= p_K^*A(K^0\pi^0) \pm q_K^*A(\bar{K}^0\pi^0) \\ &= p_K^*G_1T_1e^{i\delta_1} \pm q_K^*G_2T_2e^{i\delta_2} \\ \bar{A}(\bar{K}_{S,L}\pi^0) &= <\bar{K}_{S,L}\pi^0|\mathcal{H}|\bar{D}^0> \\ &= p_K^*\bar{A}(\bar{K}^0\pi^0) \pm q_K^*\bar{A}(K^0\pi^0) \\ &= p_K^*G_1^*T_1e^{i\delta_1} \pm q_K^*G_2^*T_2e^{i\delta_2}\end{aligned} \tag{3.29}$$

By straightforward calculation, the direct CP violation asymmetry is given as (see (3.30) on top of the next page) from $\frac{p_K}{q_K} = \frac{1+\epsilon_K}{1-\epsilon_K}$, we have $p_Kq_K^* = \frac{|q_K|^2}{|1-\epsilon_K|^2}(1-|\epsilon_K|^2 + i2Im\epsilon_K)$, then we get

$$\begin{aligned}\Delta_{K_{S,L}\pi^0} \approx \pm\, &2Im(G_1^*G_2)[sin(\delta_1-\delta_2) \\ &- 2(Im\epsilon_k)cos(\delta_1-\delta_2)]\end{aligned} \tag{3.31}$$

Since $|\epsilon_k| \sim 10^{-3}$, compare with (3.25) and (3.26), we can see that

$$\Delta_{K_s\pi^0} \approx \Delta_{K_1\pi^0}, \quad \Delta_{K_L\pi^0} \approx \Delta_{K_2\pi^0} \tag{3.32}$$

$$\Delta_{K_{S,L}\pi^0} = \frac{|A(K_{S,L}\pi^0)|^2 - |\bar{A}(\bar{K}_{S,L}\pi^0)|^2}{|A(K_{S,L}\pi^0)|^2 + |\bar{A}(\bar{K}_{S,L}\pi^0)|^2} = \frac{\pm 2Im(G_1^* G_2)Re\{p_K q_K^*[sin(\delta_1-\delta_2)+icos(\delta_1-\delta_2)]\}}{|p_K|^2|G_1|^2T_1/T_2 + |q_K|^2|G_2|^2T_2/T_1 \pm 2Re(G_1^*G_2)Re\{p_K q_K^*[cos(\delta_1-\delta_2)-isin(\delta_1-\delta_2)\}} \quad (3.30)$$

hold to a good accuracy. We indicate that since $|f> = |K_{S,L}\pi^0>$ are not CP eigenstates, therefore $|\bar{f}> \equiv CP|f> \neq \pm|f>$ and $\bar{A}(\bar{f}) \neq \bar{A}(f)$. The proper direct CP violation formula is $\Delta_f = \frac{|A(f)|^2-|\bar{A}(\bar{f})|^2}{|A(f)|^2+|\bar{A}(\bar{f})|^2}$, it is not equal to $\Delta'_f = \frac{|A(f)|^2-|\bar{A}(f)|^2}{|A(f)|^2+|\bar{A}(f)|^2}$. If using the Δ'_f formula, the result will be drastically different from (3.32) [21].

Obviously, the above discussions can be directly extended to the decay of $D^0 \to K_{S,L}X^0, (X^0 = \pi^0, \rho^0, \eta, \phi)$, we get

$$\begin{aligned} &\Delta_{K_1X^0} \approx -\Delta_{K_2X^0} = -2Jsin(\delta_1-\delta_2)_{X^0} \\ &|\Delta_{K_{1,2}X^0}| < 2J \approx 5\times 10^{-5} \end{aligned} \quad (3.33)$$

$$\begin{aligned} \Delta_{K_SX^0} &\approx \Delta_{K_1X^0} \\ \Delta_{K_LX^0} &\approx \Delta_{K_2X^0} \end{aligned} \quad (3.34)$$

3.3 The indirect CP violation

The magnitde of the indirect CP violation depends on the mixing parameter $x, y, \tilde{\epsilon}$ and ϕ_f, here $\phi_f = a + \beta_f$ (see (2.14)). Although both α and β_f are related to the phase convention, but the ϕ_f is rephasing-invariant. In this section, we will discuss the phase α which is only relevant to the $D^0 - \bar{D}^0$ mixing and the phase β_f which depends on the specific final state of the neutral D meson decays respectively, from that the magnitude of the phase ϕ_f and the indirect CP violation will be estimated.

According to the relation (2.3), we find

$$\begin{aligned} &(\frac{q}{p})^2 = |\frac{q}{p}|^2 e^{i2\alpha} = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \\ &tg(2\alpha) = \\ &-\frac{2[ReM_{12}ImM_{12} + 1/4Re\Gamma_{12}Im\Gamma_{12}]}{(ReM_{12})^2 - (ImM_{12})^2 + 1/4[(Re\Gamma_{12})^2 - (Im\Gamma_{12})^2]} \end{aligned} \quad (3.35)$$

Now we try to find the relations between ReM_{12}, ImM_{12}, $Re\Gamma_{12}$, $Im\Gamma_{12}$ and the mixing parameter Δm, $\Delta\gamma$, $\tilde{\epsilon}$. Defining $M_{12} \equiv |M_{12}|e^{i\phi_M}$, $\Gamma_{12} \equiv |\Gamma_{12}|e^{i\phi_\Gamma}$, we can obtain

$$\begin{aligned} tg\phi_M &= \frac{ImM_{12}}{ReM_{12}} \\ &= -\frac{4[Im(M_{12}\Gamma_{12}) - Im(M_{12}^*\Gamma_{12})]}{\Delta m\Delta\gamma(1+C)}, \end{aligned} \quad (3.36)$$

$$\begin{aligned} tg\phi_\Gamma &= \frac{Im\Gamma_{12}}{Re\Gamma_{12}} \\ &= -\frac{4[Im(M_{12}\Gamma_{12}) + Im(M_{12}^*\Gamma_{12})]}{\Delta m\Delta\gamma(1+C)} \end{aligned} \quad (3.37)$$

$$C = \frac{1 - tg\phi_M tg\phi_\Gamma}{1 + tg\phi_M tg\phi_\Gamma} \quad (3.38)$$

Assuming that $|ImM_{12}| \ll |ReM_{12}|$, $|Im\Gamma_{12}| \ll |Re\Gamma_{12}|$, i.e. $|tg\phi_M| \ll 1, |tg\phi_M| \ll 1$, and $C \sim 1$, and taking the limit of $\tilde{\epsilon} \to 0$, $C \to 1$, then,

$$\begin{aligned} tg2\alpha &\approx -2\frac{4\Delta m^2 tg\phi_M + \Delta\gamma^2 tg\phi_\Gamma}{4\Delta m^2 + \Delta\gamma^2} \\ &\approx (\frac{\Delta\gamma}{2\Delta m} - \frac{2\Delta m}{\Delta\gamma})\tilde{\epsilon} + \frac{4Im(M_{12}\Gamma_{12})}{\Delta m\Delta\gamma} \end{aligned} \quad (3.39)$$

Using (3.36) and (3.37), we obtain

$$\begin{aligned} tg\phi_M + tg\phi_\Gamma &\approx -\frac{4Im(M_{12}\Gamma_{12})}{\Delta m\Delta\gamma} \\ tg\phi_M - tg\phi_\Gamma &\approx \frac{(4\Delta m^2 + \Delta\gamma^2)\tilde{\epsilon}}{2\Delta m\Delta\gamma} \end{aligned} \quad (3.40)$$

$$\begin{aligned} tg2\alpha &\approx -(tg\phi_M + tg\phi_\Gamma) + (\frac{\Delta\gamma}{2\Delta m} - \frac{2\Delta m}{\Delta\gamma})\tilde{\epsilon} \\ &\approx -2tg\phi_M + \frac{\Delta\gamma}{\Delta m}\tilde{\epsilon} \approx -2tg\phi_\Gamma - \frac{4\Delta m}{\Delta\gamma}\tilde{\epsilon} \end{aligned} \quad (3.41)$$

Since $|\tilde{\epsilon}| \ll 1$ and $|tg\phi_M| \ll 1$, $|tg\phi_\Gamma| \ll 1$, we have $tg2\alpha \approx sin2\alpha \approx 2\alpha$. It follows that

$$\begin{aligned} \alpha &\approx -\phi_M + \frac{y}{x}\tilde{\epsilon} \\ &\approx -\phi_\Gamma - \frac{x}{y}\tilde{\epsilon} \\ &\approx -1/2(\phi_M + \phi_\Gamma) + 1/2(\frac{y}{x} - \frac{x}{y})\tilde{\epsilon} \end{aligned} \quad (3.42)$$

From the discussion above, we can see that although the α is relevant to the phase convention, but since $|\tilde{\epsilon}| \ll 1$ and $|ImM_{12}| \ll |ReM_{12}|$, $|Im\Gamma_{12}| \ll |Re\Gamma_{12}|$, it makes α small.

Now we discuss the phase β_f which is defined as $\rho_f \equiv |\rho_f|e^{i\beta_f}$. For f being CP eigenstates, from (3.9) we have

$$\begin{aligned} \rho_f &\equiv \frac{\bar{A}(f)}{A(f)} = \frac{\pm\bar{A}(\bar{f})}{A(f)} = \pm\frac{G_1^*T_1e^{i\delta_1} + G_2^*T_2e^{i\delta_2}}{G_1T_1e^{i\delta_1} + G_2T_2e^{i\delta_2}} \\ &= \pm\frac{1}{|A(f)|^2}[G_1^{*2}T_1^2 \\ &\quad + G_2^{*2}T_2^2 + G_1^*G_2^*2T_1T_2cos(\delta_1-\delta_2)] \end{aligned} \quad (3.43)$$

$$tg\beta_f = \frac{Im[G_1^{*2}T_1^2 + G_2^{*2}T_2^2 + G_1^*G_2^*2T_1T_2cos(\delta_1-\delta_2)]}{Re[G_1^{*2}T_1^2 + G_2^{*2}T_2^2 + G_1^*G_2^*2T_1T_2cos(\delta_1-\delta_2)]} \quad (3.44)$$

where $G_1 = V_{ud}V_{cd}^*$, $G_2 = V_{us}V_{cs}^*$, according to the Wolfenstein representation of the CKM matrix, $ReG_1 \approx$

$-ReG_2 = -\lambda(1-\frac{\lambda^2}{2})$, $ImG_1 = 0$, $-J = ImG_1^*G_2 = G_1ImG_2 = ReG_1ImG_2$, $ImG_2 = -\frac{J}{ReG_1} \approx A^2\lambda^5\eta$. It follows that

$$tg\beta_f \approx \frac{-2A^2\lambda^4\eta[T_2^2 - T_1T_2cos(\delta_1-\delta_2)]}{(1-\frac{\lambda^2}{2})[T_1^2+T_2^2-2T_1T_2cos(\delta_1-\delta_2)]} \approx -1.09\times10^{-3}F(u) \quad (3.45)$$

where we take $u = cos(\delta_1-\delta_2)(-1\le u\le 1)$ and define the function $F(u) = \frac{T_2^2-T_1T_2u}{T_1^2+T_2^2-2T_1T_2u}$, the derivative of $F(u)$ is

$$\frac{dF(u)}{du} = \frac{T_1T_2(T_2^2-T_1^2)}{(T_1^2+T_2^2-2T_1T_2u)}\begin{cases} >0\ (T_2>T_1) \\ <0\ (T_2<T_1) \\ =0\ (T_2=T_1)\end{cases} \quad (3.46)$$

(i) For the case of $T_2 > T_1$, $F(u)$ is an increasing function $F(u=-1) \le F(u) \le F(u=1)$ i.e. $\frac{1}{1+T_1/T_2} \le F(u) \le \frac{1}{1-T_1/T_2}$
(ii) For the case of $T_2 < T_1$, $F(u)$ is a decreasing function $F(u=1)\le F(u)\le F(u=-1)$ i.e. $-\frac{T_2}{T_1}\frac{1}{1-T_2/T_1} \le F(u) \le \frac{T_2}{T_1}\frac{1}{1+T_2/T_1}$ and $|F(u)| \le \frac{T_2}{T_1}\frac{1}{1-T_2/T_1}$
(iii) For the case of $T_2 = T_1$, $F(u) = 1/2$.
Therefore, for the decay of $D^0 \to K^+K^-(\frac{T_2}{T_1}\sim 10^2)$, we have $|F(u)| \approx 1$, $|tg\beta_f| \approx 1.09\times10^{-3}$. For the decay of $D^0\to\pi^+\pi^-$, $\pi^0\pi^0$, $\pi^0\rho^0$, $(\frac{T_2}{T_1}\sim10^{-2})$. We have $|F(u)| \le \frac{T_2}{T_1}$, $|tg\beta_f| \le 1.09\times10^{-5}$. While for the decay of $D^0\to K^0\bar{K}^0$, $\phi\phi(T_1=T_2)$ and $D^0\to\pi^0\eta$, $\pi^0\phi$, $\eta\eta$, $\eta\phi(T_1\sim T_2)$, we have $|F(u)|\sim O(1), |tg\beta_f|\sim10^{-3}$. It means that the phase β_f is very small in all the processes of the D^0 decays to these CP eigenstates. So we can safely write

$$|tg\beta_f| \approx |sin\beta_f| \approx |\beta_f| \le 1.09\times10^{-3} \quad (3.47)$$

For the decay of $D^0\to K_{1,2}X^0(X^0=\pi^0,\rho^0,\eta,\phi)$, in which $G_1 = V_{us}V_{cd}^*$, $G_2 = V_{ud}V_{cs}^*$, $ReG_1\approx-\lambda^2$, $ReG_2\approx(1-\lambda^2/2)^2$, $ImG_1 = 0$, $ImG_2 = -J/ReG_1 = A^2\lambda^4(1-\lambda^2/2)\eta$ and $T_1\approx T_2$, from (3.44), we get $tg\beta_f\approx-\frac{ImG_2}{ReG_2}\approx-\frac{A^2\lambda^4\eta}{(1-\lambda^2/2)}\approx-1.09\times10^{-3}$. Therefore the relation (3.47) is also satisfied in these cases.

Because α and β_f both are small, from (3.42), we obtain

$$sin\phi_f \approx \phi_f = \alpha+\beta_f \approx -\phi_M + \frac{y}{x}\tilde{\epsilon} + \beta_f \ll 1, \quad cos\phi_f \approx 1 \quad (3.48)$$

According to (3.2) the indirect CP violation asymmetry arising from the interplay between mixing and decay becomes

$$-xsin\phi_f + y\tilde{\epsilon}cos\phi_f \approx -x(-\phi_M + y/x\tilde{\epsilon}+\beta_f) + y\tilde{\epsilon} = x(\phi_M-\beta_f) \quad (3.49)$$

Because $|\phi_M| = |tg\phi_M| = |\frac{ImM_{12}}{ReM_{12}}| \sim 10^{-2}$ [22], and from (3.47), $|\beta_f|\le10^{-3}$, our conclusion is that the magnitudes of indirect CP violation asymmetries for the D^0 decays to all CP eigenstates are the same approximately which are determined by the parameter x and the phase ϕ_M

$$-x\ sin\phi_f + y\tilde{\epsilon}\ cos\phi_f \approx x\phi_M \le \sqrt{x^2+y^2}|\phi_M| \le 8.6\times10^{-4} \quad (3.50)$$
$$\mathcal{A}_{CP}(f=\pm f) \approx \Delta_f + x\phi_M$$

Where we have used $r_D = \frac{1}{2}(x^2+y^2) < 3.7\times10^{-3}$ [10]. Substituing $x\phi_M$ and differentΔ_f in Sect. 3.2 for different decay modes into (3.2), we can estimate the CP asymmetries for $D^0-\bar{D}^0$ decays into CP eigen states $\mathcal{A}_{CP}(f=\pm f)$. The values of $\mathcal{A}_{CP}(f=\pm f)$ and the lower bounds of the number of$D^0(\bar{D}^0)$ needed at 3σ level for testing CP violation are listed in Table 1.

3.4 Decays to non-CP eigenstates

For D^0 decays to non-CP eigenstates, we restrict our discussion to the cases with no direct CP violation and only one CKM factor in the decay amplitudes. It is ture for the most processes of the D^0 decays to non-CP eigenstates. Since $|\alpha|\ll1, |\bar{\theta}-\theta|\ll1$, we have

$$sin\phi^{(+)} = sin[\alpha+(\bar{\theta}-\theta)] \approx \alpha+\bar{\theta}-\theta, \quad cos\phi^{(+)}\approx1 \quad (3.51)$$

According to the formulas (3.3) and (2.36), and using the relations $\alpha y\approx-\phi_\Gamma y - x\tilde{\epsilon}$, $\alpha x\approx-\phi_M x + y\tilde{\epsilon}$, the CP asymmetry can be written as

$$\mathcal{A}_{CP}(f) = \frac{N_5}{D_5} \quad (3.52)$$

where

$$N_5 = \Big\{ |\rho_f|[x(\phi_M-\bar{\theta}+\theta)cos(\bar{\delta}_f-\delta_f) - y(\phi_\Gamma-\bar{\theta}+\theta)sin(\bar{\delta}_f-\delta_f)] - |\rho_f|^2(x^2+y^2)\tilde{\epsilon}\Big\}$$
$$D_5 = \{1+1/2|\rho_f|^2(x^2+y^2) - |\rho_f|[ycos(\bar{\delta}_f-\delta_f) + xsin(\bar{\delta}_f-\delta_f)]\}$$

For the Cabibbo favoured decay processes $D^0\to f = \{K^-\pi^+,\ K^-\rho^+,\ K^{*-}\pi^+,\ K^{*-}\rho^+,\ \bar{K}^0\pi^0,\ \bar{K}^0\rho^0,\ \bar{K}^0\eta,\ \bar{K^*}^0\phi^0,\ \bar{K^*}^0\pi^0,\ \bar{K^*}^0\rho^0,\ \bar{K^*}^0\eta\bar{K^*}^0\phi\}$, we have

$$A(f) = V_{ud}V_{cs}^*T_fe^{i\delta_f}, \quad \bar{A}(f) = V_{ud}V_{cs}^*\bar{T}_fe^{i\bar{\delta}_f} \quad (3.53)$$

where $V_{ud}V_{cs}^* = |V_{cd}V_{cs}^*|e^{i\theta}$, $V_{ud}V_{us}^* = |V_{cd}V_{us}^*|e^{i\bar{\theta}}$, and $Re(V_{ud}V_{cs}^*)\approx(1-\frac{\lambda^2}{2})^2$, $Im(V_{ud}V_{cs}^*)\approx A^2\lambda^4(1-\frac{\lambda^2}{2})\eta$,

Table 1. CP asymmetries for the neutral D meson decays to CP eigenstates f and lower bound of $N_{D^0\bar{D}^0}$ at 3σ level

$D^0 \to f$	$\mathcal{A}_{CP} = \Delta_f + x\phi_M$	h_f	upper bound of $\lvert\Delta_f\rvert$	upper bound of $\lvert\mathcal{A}_{CP}\rvert$	Br	lower bound of $N_{D^0\bar{D}^0}$
K^+K^-	$-1.09\times10^{-3}h_f sin\delta_f + x\phi_M$	$h_f \sim 10^{-2}$	$\lvert\Delta_f\rvert < 1.09\times10^{-5}$	$\lvert\mathcal{A}_{CP}\rvert < 8.6\times10^{-4}$	4.3×10^{-3}	2.8×10^{9}
$\pi^+\pi^-$					1.9×10^{-3}	8.1×10^{9}
$\pi^0\pi^0$	$-1.09\times10^{-3}\frac{1}{h_f}sin\delta_f + x\phi_M$	$h_f \sim 10^{2}$	$\lvert\Delta_f\rvert < 1.09\times10^{-5}$	$\lvert\mathcal{A}_{CP}\rvert < 8.6\times10^{-4}$	8.4×10^{-4}	1.4×10^{10}
$\pi^0\rho^0$					1.0×10^{-2}	1.2×10^{9}
$\pi^0\eta$					$< 0.4\times10^{-3}$	
$\pi^0\phi$	$-1.09\times10^{-3}\frac{sin\delta_f}{h_f+1/h_f-2cos\delta_f}$				$< 5\times10^{-4}$	
$\eta\phi$	$+x\phi_M$	$h_f \sim 0(1)$	$\lvert\Delta_f\rvert < \frac{1.09\times10^{-3}}{\lvert h_f-1/h_f\rvert}$		$< 2.8\times10^{-3}$	
$\eta\eta$					$< 7.3\times10^{-3}$	
$K^0\bar{K}^0$	$-1.09\times10^{-3}\frac{sin\delta_f}{3.66\times10^{-7}+4sin^2\frac{1}{2}\delta_f}$ $+x\phi_M$	$h_f = 1$	if $0 < \lvert sin\delta_f\rvert < 10^{-3}$ $0 < \lvert\Delta_f\rvert < 1$ (sensitive to δ_f)		1.3×10^{-3}	
$K_1\pi^0$					1.1×10^{-2}	9.9×10^{8}
$K_1\rho^0$	$-5\times10^{-5}sin\delta_f + x\phi_M$	$h_f \approx 1$	$\lvert\Delta_f\rvert < 5\times10^{-5}$	$\lvert\mathcal{A}_{CP}\rvert < 9.1\times10^{-4}$	2.7×10^{-3}	4.0×10^{9}
$K_1\eta$					6.0×10^{-3}	1.8×10^{9}
$K_1\phi$					2.9×10^{-3}	3.7×10^{9}

* indirect CP violation asymmetry (for all CP eigenstates f): $-xsin\phi_f + y\tilde{\epsilon}cos\phi_f \approx x\phi_M$ and $x|\phi_M| < 8.6\times10^{-4}$ (see text)

$$\mathcal{A}_{CP}(\bar{f}) \approx \frac{2\lambda^2 h_f[x\phi_M cos(\bar{\delta}_f-\delta_f) + y\phi_\Gamma sin(\bar{\delta}_f-\delta_f)] - 2(x^2+y^2)\tilde{\epsilon}}{2\lambda^4 h_f^2 + (x^2+y^2) - 2\lambda^2 h_f[ycos(\bar{\delta}_f-\delta_f) - xsin(\bar{\delta}_f-\delta_f)]} \tag{3.59}$$

$ReV_{cd}V_{us}^* \approx -\lambda^2$, $Im(V_{cd}V_{us}^*) = 0$, it follows that

$$\begin{aligned} tg\theta &= \frac{Im(V_{ud}V_{cs}^*)}{Re(V_{ud}V_{cs}^*)} = \frac{A^2\lambda^4\eta}{(1-\lambda^2/2)} \approx 5\times10^{-4}, \\ \theta &\approx tg\theta \approx 5\times10^{-4} \approx 0 \\ tg\bar{\theta} &= \frac{Im(V_{cd}V_{us}^*)}{Re(V_{cd}V_{cs}^*)} = 0 \\ \bar{\theta} &= 0 \\ |\rho_f| &= |\frac{\bar{A}(f)}{A(f)}| = |\frac{V_{cd}V_{us}^*\bar{T}_f}{V_{ud}V_{cs}^*T_f}| \approx \frac{\lambda^2}{(1-\lambda^2/2)^2}\frac{\bar{T}_f}{T_f} \approx \lambda^2 h_f \\ &\ll 1 \quad (h_f \equiv \frac{\bar{T}_f}{T_f} \sim O(1)) \end{aligned} \tag{3.54}$$

From (3.52) and (3.54), the CP asymmetry can be simplified to

$$\begin{aligned} \mathcal{A}_{CP}(f) &\approx \lambda^2 h_f[x\phi_M cos(\bar{\delta}_f-\delta_f) - y\phi_\Gamma sin(\bar{\delta}_f-\bar{\delta}_f)] \\ &< \lambda^2 h_f\sqrt{(x\phi_M)^2 + (y\phi_\Gamma)^2} \end{aligned} \tag{3.55}$$

Using the relations $\phi_M = -\alpha + \frac{y}{x}\tilde{\epsilon}$, $\phi_\Gamma = -\alpha - \frac{x}{y}\tilde{\epsilon}$,we have

$$\begin{aligned} (x\phi_M)^2 + (y\phi_\Gamma)^2 &= (x^2+y^2)(\alpha^2+\tilde{\epsilon}^2) \\ &\approx (x^2+y^2)\alpha^2 \end{aligned} \tag{3.56}$$

So the upper bound of the CP asymmetry is

$$\mathcal{A}_{CP}(f) \leq \lambda^2 h_f|\alpha|\sqrt{x^2+y^2} \tag{3.57}$$

Taking $\lambda^2 \approx 0.05$, $h_f \approx 1$, $|\alpha| \approx |\phi_M| \sim 10^{-2}$, $x^2+y^2 < 7.4\times10^{-3}$, we get $\mathcal{A}_{CP}(f) < 4.3\times10^{-5}$.

For the double Cabibbo suppressed decay processes $D^0 \to \bar{f} = \{K^+\pi^-, K^+\rho^-, K^{*+}\pi^-, K^{*+}\rho^-, K^0\pi^0, K^0\rho^0, K^0\eta, K^0\phi, K^{*0}\pi^0, K^{*0}\rho^0, K^{*0}\eta, K^{*0}\phi\}$, we have

$$\begin{aligned} A(\bar{f}) &= V_{cd}^*V_{us}T_{\bar{f}}e^{i\delta_{\bar{f}}} = (V_{cd}V_{us}^*)^*\bar{T}_f e^{i\bar{\delta}_f} \\ \bar{A}(\bar{f}) &= V_{ud}^*V_{cs}\bar{T}_{\bar{f}}e^{i\bar{\delta}_{\bar{f}}} = (V_{ud}V_{cs}^*)^*T_f e^{i\delta_f} \end{aligned} \tag{3.58}$$

i.e. $(\bar{\theta}-\theta)_{\bar{f}} = (\theta-\bar{\theta})_f \approx 5\times10^{-4} \approx 0$, $\delta_{\bar{f}} = \bar{\delta}_f$ $\bar{\delta}_{\bar{f}} = \delta_f$, $|\rho_{\bar{f}}| = 1/|\rho_f| \approx \frac{1}{\lambda^2 h_f}$.

From (3.52), we obtain (see (3.59) under Table 1). The upper bound of the CP asymmetry is

$$\begin{aligned} \mathcal{A}_{CP}(\bar{f}) &\leq \frac{2\lambda^2 h_f\sqrt{\alpha^2+\tilde{\epsilon}^2}\sqrt{x^2+y^2} - 2(x^2+y^2)\tilde{\epsilon}}{2\lambda^4 h_f^2 + (x^2+y^2) - 2\lambda^2 h_f\sqrt{x^2+y^2}} \\ &\approx \frac{2\lambda^2 h_f|\alpha|\sqrt{x^2+y^2}}{\lambda^4 h_f^2 + (\lambda^2 h_f - \sqrt{x^2+y^2})^2} \end{aligned} \tag{3.60}$$

Also taking $\lambda^2 = 0.05$, $h_f = 1$, $|\alpha| = |\phi_M| \sim 10^{-2}$, $x^2+y^2 < 7.4\times10^{-3}$, we get $\mathcal{A}_{CP}(\bar{f}) < 2.3\times10^{-2}$. Although the DCSD branching ratio $B_r(\bar{f})$ is small

$$\frac{Br(\bar{f})}{Br(f)} \approx \frac{|V_{us}V_{cd}^*|^2}{|V_{ud}V_{cs}^*|^2} \approx 2.5\times10^{-3} \tag{3.61}$$

$$\mathcal{A}^-_{CP}(f_1,f_2) = \frac{2(1-|\rho_{f_1}|^2|\rho_{f_2}|^2)(x^2+y^2)\tilde{\epsilon} + 4|\rho_{f_1}||\rho_{f_2}|sin[(\bar{\theta}_{f_1}-\theta_{f_1})-(\bar{\theta}_{f_2}-\theta_{f_2})]sin[(\bar{\delta}_{f_1}-\delta_{f_1})-(\bar{\delta}_{f_2}-\delta_{f_2})]}{(1+|\rho_{f_1}|^2|\rho_{f_2}|^2)(x^2+y^2) - 4|\rho_{f_1}||\rho_{f_2}|cos[(\bar{\theta}_{f_1}-\theta_{f_1})-(\bar{\theta}_{f_2}-\theta_{f_2})]cos[(\bar{\delta}_{f_1}-\delta_{f_1})-(\bar{\delta}_{f_2}-\delta_{f_2})]} \quad (3.63)$$

$$\mathcal{A}^-_{CP}(f_1,f_2) = \frac{2(1-|\rho_{f_1}|^2|\rho_{f_2}|^2)(x^2+y^2)\tilde{\epsilon}}{(1+|\rho_{f_1}|^2|\rho_{f_2}|^2)(x^2+y^2) - 4|\rho_{f_1}||\rho_{f_2}|cos[(\bar{\delta}_{f_1}-\delta_{f_1})-(\bar{\delta}_{f_2}-\delta_{f_2})} \quad (3.64)$$

$$\mathcal{A}^-_{CP}(f_1,f_2) \approx \frac{2(1-h_{f_1}h_{f_2})(x^2+y^2)\tilde{\epsilon}}{(1+h^2_{f_1}h^2_{f_2})(x^2+y^2) - 4h_{f_1}h_{f_2}cos[(\bar{\delta}_{f_1}-\delta_{f_1})-(\bar{\delta}_{f_2}-\delta_{f_2})]} \quad (3.67)$$

$$\mathcal{A}^-_{CP}(f_1=\bar{f}_1,f_2) = \frac{(1-|\rho_{f_2}|^2)[-\Delta_{f_1}+(x^2+y^2)\tilde{\epsilon}] - 2|\rho_{f_2}|sin[\beta_{f_1}-(\bar{\theta}_{f_2}-\theta_{f_2})]sin(\bar{\delta}_{f_2}-\delta_{f_2})}{1+|\rho_{f_2}|^2 - 2|\rho_{f_2}|cos[\beta_{f_1}-(\bar{\theta}_{f_2}-\theta_{f_2})]cos(\bar{\delta}_{f_2}-\delta_{f_2})} \quad (3.68)$$

but the number of $D^0 - \bar{D}^0$ pairs needed for testing the CP asymmetry at 3σ level is $N_{D\bar{D}} \sim \frac{9}{Br\cdot\mathcal{A}^2_{CP}}$, we find

$$\frac{N_{D\bar{D}}(f)}{N_{D\bar{D}}(\bar{f})} \approx \frac{Br(\bar{f})}{Br(f)}\left(\frac{\mathcal{A}_C(\bar{f})}{\mathcal{A}_C(f)}\right)^2 \approx 7.2\times 10^2 \quad (3.62)$$

It means that we need more $D^0 - \bar{D}^0$ pairs for Cabbibo favoured decays than for Double Cabbibo suppressed decays. So we see that, for example, the double Cabibbo suppressed decay $D^0 \to K^+\pi^-$ is superior compared with the Cabibbo favoured decay $D^0 \to K^-\pi^-$ for testing CP violation.

The numerical results of CP asymmetries and the number of $D^0 - \bar{D}^0$ pairs needed for testing CP violation at 3σ level are listed in Table 2.

3.5 Coherent $D^0\bar{D}^0$ pair decays

According to our discussion in Sect. 3.1, there are an important simplification for decay processes of the coherent $(D^0\bar{D}^0)$ pair with semileptonic tagging. In experiment, we can use the decay processes of the coherant $(D^0\bar{D}^0)_-$ pair with the semileptonic tagging to measure the mixing parameter $\tilde{\epsilon}$, the direct and indirect CP violation (see (3.6), (3.7) and (3.8)) respectively. In this section, we discuss the case of the coherent $(D^)\bar{D}^0)_-$ pair decays in which, each D^0 or $\bar{D}^0$ decays into two mesons.

For the case of f_1 and f_2 being non-CP eigenstates from (3.4) and (2.36) the CP asymmetry becomes (see (3.63) on top of the page).

Letting $\{K^-\pi^+,\dots\}$ represente the Cabibbo favoured decay processes, and $\{K^+\pi^-,\dots\}$ represente the double Cabibbo suppressed decay processes. Since $\mathcal{A}^-_{CP}(\bar{f}_1\bar{f}_2) = -\mathcal{A}^-_{CP}(f_1f_2)$ and $\mathcal{A}^-_{CP}(\bar{f}_1f_2) = -\mathcal{A}^-_{CP}(f_1\bar{f}_2)$, we only need to discuss the two cases: (i) $f_1, f_2\epsilon\{K^+\pi^-,\dots\}$, and (ii) $f_1\epsilon\{K^+\pi^-,\dots\}$, $f_2\epsilon\{K^-\pi^+,\dots\}$. Since $\bar{\theta}_{f_1}-\theta_{f_1} = \bar{\theta}_{f_2}-\theta_{f_2}$ for all the cases, then the formula (3.71) can be written as (see (3.64) on top of the page).

(i) $f_1, f_2\epsilon\{K^+\pi^-,\dots\}$ then $|\rho_{f_1}| \approx \lambda^2 h_{f_1} \ll 1$, $|\rho_{f_2}| \approx \lambda^2 h_{f_2} \ll 1$, so we have

$$\mathcal{A}^-_{CP}(f_1,f_2) \approx \frac{2(x^2+y^2)\tilde{\epsilon}}{(x^2+y^2) - 4\lambda^4 h_{f_1}h_{f_2}cos[(\bar{\delta}_{f_1}-\delta_{f_1})-(\bar{\delta}_{f_2}-\delta_{f_2})]} \quad (3.65)$$

For the case of $f_2 = f_1$ and taking $\lambda^2 = 0.05$, $h_{f_1} = 1$, $x^2+y^2 = 7.4\times10^{-5}$ we get

$$\mathcal{A}^-_{CP}(f_1,f_1) \approx \frac{2\tilde{\epsilon}}{1-4\lambda^4 h^2_{f_1}/(x^2+y^2)} \approx -5.7\tilde{\epsilon} \quad (3.66)$$

(ii) $f_1\epsilon\{K^-\pi^+,\dots\}$, $f_2\epsilon\{K^+\pi^-,\dots\}$, then $|\rho_{f_1}| \approx \lambda^2 h_{f_1}$, $|\rho_{f_2}| \approx \frac{1}{\lambda^2}h_{f_2}$, so we have (see (3.67) on top of the page). For the case of $f_2 = \bar{f}_1$ since $h_{\bar{f}_1} = 1/h_{f_1}$. we obtain $\mathcal{A}^-(f,\bar{f}) = 0$, it means, for examples, $\Gamma^-(K^-\pi^+,\ K^+\pi^-) = \Gamma^-(K^+\pi^-,\ K^-\pi^+)$.

For the case of f_1 being CP eigenstates and f_2 non eigenstates from (3.5) and (2.36), we have (see (3.68) on top of the page).

Since $\mathcal{A}^-_{CP}(f_1=\bar{f}_1,\bar{f}_2) = -\mathcal{A}^-_{CP}(f_1=\bar{f}_1,f_2)$, assuming $f_2\epsilon\{K^-\pi^+,\dots\}$, $|\rho_{f_2}| \approx \lambda^2 h_{f_2} \ll 1$, from (3.68) we get

$$\mathcal{A}^-_{CP}(f_1=\bar{f}_1,f_2) = -\Delta_{f_1} + (x^2+y^2)\tilde{\epsilon} - 2\lambda^2 h_{f_2} \times sin[\beta_{f_1}-(\bar{\theta}_{f_2}-\theta_{f_2})]sin(\bar{\delta}_{f_2}-\delta_{f_2}) \quad (3.69)$$

In (3.69), the first term Δ_{f_1} represents the direct CP violation for decay to CP eigenstates f_1 which have been discussed in Sect. 3.2. The second term $(x^2+y^2)\tilde{\epsilon}$ arises from the mixing which only depends on the mixing parameter and assuming $|\tilde{\epsilon}| \sim 10^{-3}$, $x^2+y^2 < 7.4\times10^{-3}$, it is very small, i.e. $(x^2+y^2)|\tilde{\epsilon}| < 7.4\times10^{-6}$. The third term describes indirect CP violation caused by decay only. Since $|\beta_{f_1}| < 10^{-3}$, $|\bar{\theta}_{f_2}-\theta_{f_2}| = 5\times10^{-4}$, then $|sin[\beta_{f_1} - (\bar{\theta}_{f_2}-\theta_{f_2})]| < 10^{-3}$. Taking $\lambda^2 = 0.05$, $h_{f_2} = 1$, we find

$$|2\lambda^2 h_{f_2} sin[\beta_{f_1}-(\bar{\theta}_{f_2}-\theta_{f_2})]sin(\bar{\delta}_{f_2}-\delta_{f_2})| < 10^{-4}|sin(\bar{\delta}_{f_2}-\delta_{f_2})| \quad (3.70)$$

One can see that if mixing parameter vanishes, there may still exists indirect CP violation, but it is small.

4 Conclusions and discussions

In this paper, the generic time-integrated CP asymmetry formulas for the decay of the $D^0 - \bar{D}^0$ systems are presented without specific assumption. Using these formulas,

Table 2. The CP asymmetries for the neutral D meson decays to the non-CP eigenstates and the lower bound of $N_{D^0\bar{D}^0}$ at 3σ level, where $\lambda^2 = 0.05$, $h_f = 1$ $|\alpha| \simeq |\phi_M| \sim 10^{-2}$, $x^2 + y^2 < 7.4 \times 10^{-3}$

$D \to f$:	$K^-\pi^+$	$K^-\rho^+$	$K^{*-}\pi^+$	$K^{*-}\rho^+$	$\bar{K}^0\pi^0$	$\bar{K}^0\rho^0$	$\bar{K}^0\eta$	$\bar{K}^0\phi$	$\bar{K}^{*0}\pi^0$	$\bar{K}^{*0}\rho^0$	$\bar{K}^{*0}\eta$	$\bar{K}^{*0}\phi$
$Br(\%)$:	3.8	10.8	5.0	6.0	2.1	1.2	0.70	0.85	3.1	1.5	1.9	1.1
$\lvert\mathcal{A}_{CP}(f)\rvert < \lambda^2 h_f \lvert\alpha\rvert \sqrt{x^2+y^2} \approx 4.3 \times 10^{-5}$												
lower bound of $N_{D^0\bar{D}^0}(f)$:	1.3×10^{11}	4.5×10^{10}	9.9×10^{10}	8.1×10^{10}	2.3×10^{11}	4.1×10^{11}	6.9×10^{11}	5.8×10^{11}	1.5×10^{11}	3.2×10^{11}	2.5×10^{11}	4.4×10^{11}
$D \to \bar{f}$:	$K^+\pi^-$	$K^+\rho^-$	$K^{*+}\pi^-$	$K^{*+}\rho^-$	$K^0\pi^0$	$K^0\rho^0$	$K^0\eta$	$K^0\phi$	$K^{*0}\pi^0$	$K^{*0}\rho^0$	$K^{*0}\eta$	$K^{*0}\phi$
$Br(\%)$:	9.5×10^{-3}	2.7×10^{-2}	1.3×10^{-2}	1.5×10^{-2}	5.3×10^{-3}	3.0×10^{-3}	1.8×10^{-3}	2.1×10^{-3}	7.8×10^{-3}	3.8×10^{-3}	4.8×10^{-3}	2.8×10^{-3}
$\lvert\mathcal{A}_{CP}(\bar{f})\rvert < 2\lambda^2 h_f \sqrt{x^2+y^2}/[\lambda^4 h_f^2 + (\lambda^2 h_f - \sqrt{x^2+y^2})^2] \approx 2.3 \times 10^{-2}$												
lower bound of $N_{D^0\bar{D}^0}(\bar{f})$:	1.7×10^{8}	6.2×10^{7}	1.4×10^{8}	1.2×10^{8}	3.2×10^{8}	5.7×10^{8}	9.9×10^{8}	8.0×10^{8}	2.2×10^{8}	4.5×10^{8}	3.5×10^{8}	6.1×10^{8}

we systematically discussed the $D^0 - \bar{D}^0$ mixing and CP violation for neutral D meson decays both incoherently and coherently to a veriety of CP eigenstates and non-CP eigenstates. The direct CP violation Δ_f and the important rephasing-invariant parameter for the indirect CP violation $\phi_f = \alpha + \beta_f$ are analysed in detail and the upper bound for the various CP violating terms (direct, mixing, interplay of mixing and decay) are given using extremum method to eliminate the unknown strong phase-shift. The results are listed in Table 1 and 2.

For the case of f being CP eigenstates, from Table 1, it can be seen that for the decay processes where $h_f >> 1$ or $h_f \ll 1$ the direct CP violation asymmetry is very small, $|\Delta_f| < 10^{-5}$, and for the case of $h_f \sim O(1)$, $|\Delta_f| < 10^{-4} \sim 10^{-3}$, while for the case of $h_f = 1$, the direct CP violation asymmetry is very sensitive to the strong phase-shift and could become quite large. So the most promising decay channels to observe the large direct CP violation might be $D^0 \to K^0\bar{K}^0, \phi\phi$, if the strong phase-shift $sin\delta_f \neq 0$. On the other hand although the $K_{S,L}X^0 (X^0 = \pi^0, \rho^0, \eta, \phi)$ are not the exact CP eigenstates due to the existence of small CP violation in $K^0 - \bar{K}^0$ system, but according to our CP asymmetry definition $\Delta_f = \frac{|A(f)|^2 - |\bar{A}(\bar{f})|^2}{|A(f)|^2 + |\bar{A}(\bar{f})|^2}$, the magnitude of the direct CP violation for $D^0 \to K_{S,L}X^0$ is approximately equal to the magnitude for D^0 decays to the CP eigenstates $K_{1,2}X^0$ which is smaller than 10^{-4}. This means that the effects of the $K^0 - \bar{K}^0$ mixing in the final state of the neutral D meson decays to $K_{S,L}X^0$ is negligible (see (3.31)).

From our discussion in Sect. 3.3, since $|\beta_f| < 10^{-3}$, $\phi_f = \alpha + \beta_f \approx \alpha$ (if $\alpha \sim 10^{-2}$), then the indirect CP violation asymmetries of the neutral D meson decays to CP eigenstates are approximately the same for all the decay modes and equal to $x\phi_M$. Different from the direct CP violation, the indirect CP violation does not depend on the decay modes approximately while mainly depends upon the $D^0 - \bar{D}^0$ mixing parameter. This feature is useful for experimental searches.

For the case of f being non-CP eigenstates, from Table 2, we can see that the indirect CP violation asymmetry of the Cabibbo favoured decay processes is very small ($< 10^{-5}$) while it could be large for the double Cabibbo suppressed decay processes. Although the branching ratio is small the double Cabibbo suppressed decay processes are still superior to the Cabibbo favoured decay processes for testing the CP violation in experiment. For the decay processes $D^0 \to K^+\rho^-, K^{*+}\rho^-, K^+\pi^-$ one needs about 10^8 $D^0\bar{D}^0$ events to test the CP asymmetry. It should be achievable in the τ-charm factory [23].

Usually direct CP violation asymmetry in transition amplitudes and indirect CP violation arising from the interplay of mixing and decay appear simultaneously for the case of $D^0(\bar{D}^0)$ decays to the CP eigenstates, but in the decay processes of the $D^0(\bar{D}^0)_-$ pair with semileptonic tagging because of the coherente we can distinguish them from one another by the relations $\mathcal{A}^{(-)}_{CP}(f, \ell^+X^-) \approx -\Delta_f + (x^2+y^2)\tilde{\epsilon}$ and $\mathcal{A}_{CP}(f) + \mathcal{A}^{(-)}_{CP}(f, \ell^+X^-) \approx -xsin\phi_f + y\tilde{\epsilon}cos\phi_f \approx x\phi_M$ (see (3.7), (3.8) and (3.58)), while for f being non-CP eigenstates, we have more simple relation $\mathcal{A}^{(-)}_{CP}(f, \ell^+X^-) = 2\tilde{\epsilon}$. Since the CP asymmetry $\mathcal{A}^{(-)}_{CP}(f, \ell^+X^-)$ (also for $\mathcal{A}^{(-)}_{CP}(f, \ell^-X^+) = -\mathcal{A}^{(-)}_{CP}(f, \ell^+X^-)$ is independent of the decay final-state f, it will increase the number of decay events in statistics. If $\tilde{\epsilon} \sim 10^{-3}$, such a CP violating signal might be detectable in experiment. Conversely we can use this relation to measure the mixing parameter $\tilde{\epsilon}$. We also discussed various cases for coherent $(D^0\bar{D}^0)_-$ pair in which each D^0 or $\bar{D}^0$ decay to two mesons. Since the coherent $(D^0\bar{D}^0)_-$ pair can be producted by $e^+e^- \to \psi(3.77) \to (D^0\bar{D}^0)_-$ or $e^+e^- \to \Psi(4.14) \to (D^{*0}\bar{D}^0)_- \to \pi^0(D^0\bar{D}^0)_-$, so these results are specially useful for the programs of the proposed τ-charm factories.

We note that although the upper bounds of various CP violation asymmetries for decays of the $D^0 - \bar{D}^0$ system given in this paper, are useful for searching CP violation signal, testing the standard model and probing new physics beyond the standard model, in order to give more precise numerical predictions for CP asymmetries, it needs to take more efforts to calculate the accurate magnitude of $h_f = T_1/T_2$ (specially in the case of $h_f \sim O(1)$) and strong phase-shift $\delta_f = \delta_1 - \delta_2$.

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